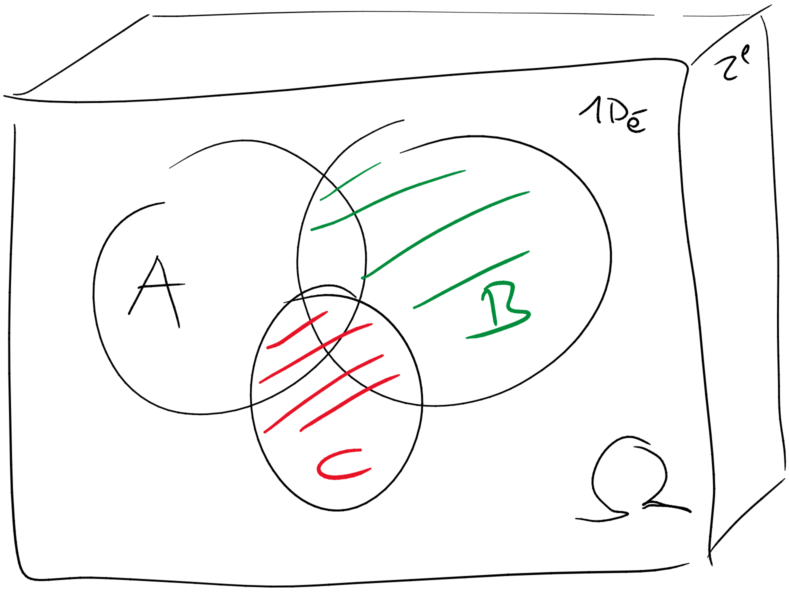
Cours 10.05.2021 Matin

lundi, 10 mai 2021 10:21



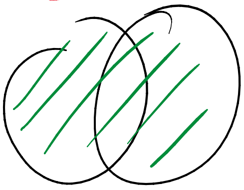
$\mathbb{P}(A)$ $\quad A \subseteq \Omega \quad$ $P(\Omega) = 1$!

$$C_p^n = \frac{n!}{p!(n-p)!}$$

$A \cup B \Rightarrow A$ ou B (inclusif)

$A \cap B \Rightarrow A$ et B

$$\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)}$$



$A \cup B$

Dé 1: Ω_1 : Lancé du premier dé $x \in \{1, 2, 3, 4, 5, 6\} \rightarrow x_1$

Dé 1 : Ω_1 : Lancé du premier dé $x \in \{1, 2, 3, 4, 5, 6\}$ $\quad X_1$

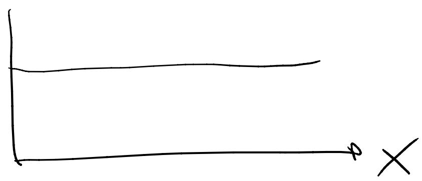
<u>Dé 2</u> : Ω_2 : $\quad x_2 \in \{1, \ldots, 6\}$ $\quad X_2$

Variables Aléatoires

$$\mathbb{P}(X_1 + X_2) = \mathbb{P}(X_1) + \mathbb{P}(X_2)$$ (⚠ X_1 et X_2 sont indépendants)

$$X_{\text{somme de dés}} = X_{\text{Dé 1}} + X_{\text{dé 2}}$$

X suit Loi uniforme

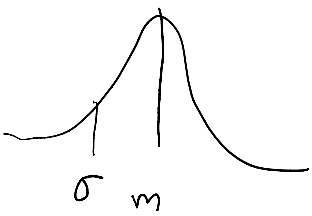


Loi Normale

Loi Exponentielle

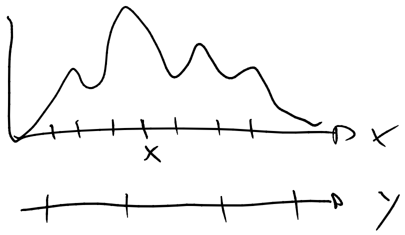
Binomiale, Bernoulli, Poisson, ...

$X \sim$



$X \longrightarrow Y = c\cdot X + d$ $\quad c,d \in \mathbb{R}$ constante

$\mathbb{P}(y_i) = \mathbb{P}(c\cdot x + d)$



$\mathbb{P}(c\cdot x) \neq c\cdot \mathbb{P}(x)$

$\mathbb{P}(c + x) \neq c + \mathbb{P}(x)$

Exemple:

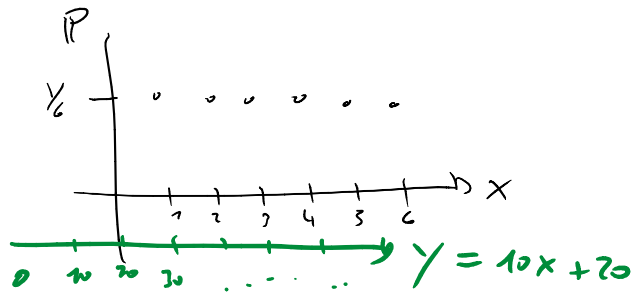
Mesurons X = lancé d'un dé

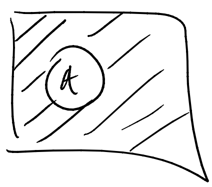
$Y = 10\cdot x$ (10 x la valeur du lancé)

X		$\mathbb{P}$
1	10	1/6
2	20	1/6
3	30	.
4	40	.
5	50	.
6	60	.

$\mathbb{P}(X=1) = \frac{1}{6}$

$\mathbb{P}(Y=1) = 0 \qquad \mathbb{P}(Y=10) = \frac{1}{6}$





$\mathbb{P}(\Omega) = 1$

$$\mathbb{P}(\Omega\setminus A) = 1 - \mathbb{P}(A)$$

$$\mathbb{P}(\emptyset) = 0.$$

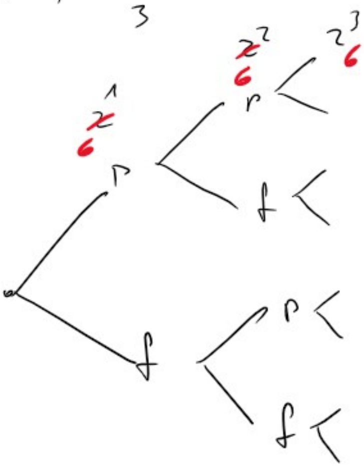
Lancé de pièce : $\mathbb{P}(\text{pile}) = \frac{2}{3}$ $\quad \mathbb{P}(\text{face}) = \frac{1}{3}$

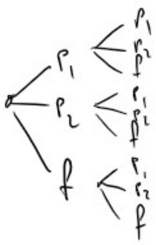
$\mathbb{P}(\text{pile et pile}) = \ ? = \frac{2}{3} \times \frac{2}{3}$

$\mathbb{P}(\text{pile puis face}) = \ ? = \frac{2}{3} \times \frac{1}{3}$

$\mathbb{P}(\text{face - pile}) = \frac{1}{3} \times \frac{2}{3}$

$\mathbb{P}(f - f) = \frac{1}{3} \times \frac{1}{3}$





	$\mathbb{P}$
$p_1 p_1$	$1/9$
$p_1 p_2$	$1/9$
$p_1 f$	$1/9$
$p_2 p_1$	.
$p_2 p_2$	.
$p_1 f$	.
$f p_1$	.
$f p_2$	.
ff	.

$\xrightarrow{p_1 = p_2}$

$pp = \frac{4}{9} = \frac{2}{3} \times \frac{2}{3} = \mathbb{P}(p) \times \mathbb{P}(p)$

$pf = \frac{2}{9} = \frac{2}{3} \cdot \frac{1}{3} = \mathbb{P}(p) \cdot \mathbb{P}(f)$

$fp = \frac{2}{9} \cdot \quad = \mathbb{P}(f) \cdot \mathbb{P}(p)$

$ff = \frac{1}{9} = \mathbb{P}(f) \cdot \mathbb{P}(f)$

$$\mathbb{P}(f\,p\,f\,f\,p) = \mathbb{P}(f) \times \mathbb{P}(p) \times \mathbb{P}(f)^2 \times \mathbb{P}(p) = \frac{1}{3}^3 \times \frac{2}{3}^2 = \frac{4}{243}$$

$\left.\begin{array}{l} p = 1 \\ f = 0 \end{array}\right\} \sum_{3\text{ lancés}} \text{piles} \qquad \Omega = \{0, 1, 2, 3\} = Y$

$\mathbb{P}(\Omega) = 1$

$\mathbb{P}(0) = \mathbb{P}(f)^3 = \frac{1}{27} = \frac{1}{27} \quad C_1^3$

$\mathbb{P}(1) = 3\,|\mathbb{P}(p) \times \mathbb{P}(f)^2| = \frac{2}{\ } = \frac{6}{\ }$

$$\mathbb{P}(1) = 3\left(\mathbb{P}(p) \times \mathbb{P}(f)^2\right) = 3 \cdot \frac{2}{27} = \frac{6}{27}$$

$$\mathbb{P}(2) = 3\,\mathbb{P}(p)^2 \times \mathbb{P}(f) = 3\,\frac{4}{27} = \frac{12}{27}$$

$$\mathbb{P}(3) = \mathbb{P}(p)^3 = \frac{8}{27} = \frac{8}{27}$$

$$\frac{15}{27} \qquad \frac{27}{27} = 1$$

Exercice : quelle est la probabilité de gagner au loto suisse

On tire 6 boules parmi 42 puis 1 boule parmi 6

C_6^{42} = nombre de combinaisons possibles

$x_1\, x_2\, x_3\, x_4\, x_5\, x_6$

$\frac{1}{42} \quad \frac{1}{41}$

$$\mathbb{P}(x_1 x_2 \ldots x_6) = \frac{1}{42 \cdot 41 \cdot \ldots \cdot 37} = \frac{1}{3 \cdot 10^9}$$

Si $x_1 \ldots x_6$ gagnant

$x_2 x_5 x_4 \ldots x_1$ gagnant aussi

↳ toutes les permutations sont gagnantes

combien y en a-t-il ? ⇒ $6!$

$$\mathbb{P}(\text{Gagner } 6\text{ n°}) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{42 \cdot 41 \cdot \ldots \cdot 37} = \frac{1}{5'245'786}$$

$$\mathbb{P}(\text{Gagner } 6+1) = \mathbb{P}(6) \cdot \mathbb{P}(\text{compl}) = \frac{1}{31 \cdot 10^6}$$